

DC Motor

A dc motor is similar in construction to a dc generator. As a matter of fact a dc generator will run as a motor when its field & armature windings are connected to a source of direct current.

The basic construction is same whether it is generator or a motor.

Working principle : The principle of operation of a dc motor can be stated as when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force. In a practical dc motor, the field winding produces the required magnetic field while armature conductor play the role of current carrying conductor and hence the armature conductors experience a force.

As conductors are placed in the slots which are on the periphery, the individual force experienced by the conductive acts as a twisting or turning force on the armature which is called a torque.

The torque is the product of force and the radius at which this force acts, so overall armature experiences a torque and starts rotating.

Consider a single conductor placed in a magnetic field , the magnetic field is produced by a permanent magnet but in practical dc motor it is produced by the field winding when it carries a current.

Now this conductor is excited by a separate supply so that it carries a current in a particular direction.

Consider that it carries a current away from an current. Any current carrying conductor produces its own magnetic field around it, hence this conductor also produces its own flux, around. The direction of this flux can be determined by right hand thumb rule. For direction of current considered the direction of flux around a conductor is clock-wise. Now, there are two fluxes present

1. Flux produced by permanent magnet called main flux
2. Flux produced by the current carrying conductor

From the figure shown below, it is clear that on one side of the conductor, both the fluxes are in the same direction in this case, on the left of the conductor there gathering of the flux lines as two fluxes help each other. As to against this, on the right of the conductor, the two fluxes are in opposite direction and hence try to

cancel each other. Due to this, the density of the flux lines in this area gets weakened.

So on the left, there exists high flux density area while on the right of the conductor then exists low flux density area as shown.

The flux distribution around the conductor acts like a stretched ribbed band under tension. It exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area, i.e. from left to right from the case considered as shown above.

In the practical dc motor, the permanent magnet is replaced by the field winding which produces the required flux. The field winding produces the required flux called main flux and all the armature conductors, which are on the periphery of the armature, get subjected to the mechanical force.

Due to this, overall armature experiences a twisting force called torque and armature of the motor starts rotating.

Direction of rotation of motor

The magnitude of the force experienced by the conductor in a motor is given by $F = BIL$ newtons.

The direction of the main field can be reversed by changing the direction of current passing through the field winding, which is possible by interchanging the polarities of supply which is given to the field winding.

The direction of current through armature can be reversed by changing supply polarities of dc supplying current to the armature.

If directions of both the currents are changed then the direction of rotation of the motor remains undisturbed.

In a dc motor both the field and armature are connected to a source of direct current. The current through the armature winding establishes its own magnetic flux. The interaction between the main field and the armature current produces the torque, thereby causing the motor to rotate. Once the motor starts rotating, already existing magnetic flux there will be an induced emf in the armature conductors due to generator action. This emf acts in a direction opposite to supplied voltage. Therefore it is called Back emf.

Significance of Back emf

In the generating action, when a conductor cuts the lines of flux, emf gets induced in the conductor in a motor, after a motoring action, armature starts rotating and armature conductors cut the main flux.

After a motoring action, there exists a generating action there is induced emf in the rotating armature conductors according to Faraday's law of electromagnetic induction. This induced emf in the armature always acts in the opposite direction of the supply voltage. This is according to the Lenz's law which states that the direction of the induced emf is always so as to oppose the cause producing it.

In a dc motor, electrical input i.e., the supply voltage is the cause and hence this induced emf opposes the supply voltage.

The emf tries to set up a current throughout the armature which is in the opposite direction to that which supply voltage is forcing through the conductor so, as this emf always opposes the supply voltage, it is called back emf and denoted as E_b .

Through it is denoted as E_b , basically it gets generated by the generating action which we have seen earlier So, $E_b = \frac{\phi ZNP}{60A}$

Voltage equation of a Motor

The voltage V applied across the motor armature has to (1) overcome the back emf E_b and

(1) supply the armature ohmic drop $I_a R_a$

$$V = E_b + I_a R_a$$

This is known as voltage equation of a motor

Multiplying both sides by I_a , we get

$$VI_a = E_b I_a + I_a^2 R_a$$

VI_a = electrical input to the armature

$E_b I_a$ = electrical equivalent of mechanical Power developed in the armature

$I_a^2 R_a$ = ohmic loss in the armature

Hence, out of the armature input, some is wasted in $I_a^2 R$ loss and the rest is converted into mechanical power within the armature.

Motor efficiency is given by the ratio of power developed by the armature to its input i.e. $E_b I_a / VI_a = E_b / V$.

Higher the value of E_b as compared to v , higher the motor efficiency.

Conduction for maximum powers

The gross mechanical developed by a motor = $p_m = vI_a - I_a^2 R_a$

$$\frac{dP_m}{dI_a} \quad v - 2I_a R_a \quad I_a R_a = v/2$$

$$\text{As } v = E_b + I_a R_a \quad \text{and } I_a R_a = v/2 \quad E_b = v/2$$

Thus gross mechanical power developed by a motor is maximum when back emf is equal to half the applied voltage. This conduction's how ever at realized in practice, because in that case current will be much beyond the normal current of the motor.

More ova, half the input would be wasted in the form of heat and taking other losses into consideration the motor efficiency will be well below 50 %.

1. A 220v – dc machine has an armature resistance of 0.5 Ω . If the full road armature current is 20A, find the induced emf when the machine acts (1) generator (2) motor.

The dc motor is assumed to be shunt connected in cash case, short current in considered negligible because its value is not given.

$$(a) \text{ As generator } E_g = v + I_a R_a = 220 + 0.5 \times 20 = 230 \text{ v}$$

$$(b) \text{ As motor } E_b = v - I_a R_a = 220 - 0.5 \times 20 = 210 \text{ v}$$

2. A 440 v, shunt motor has armature resistance of 0.8 a and field resistance of 200 Ω . Determine the back emf when giving an output at 7.46 kw at 85% efficiency.

$$\text{Motor input power} = \frac{7.46 \times 10^3}{0.85} \text{ w}$$

$$\text{Motor input current} = \frac{7460}{0.85 \times 440} = 19.95 \text{ A}$$

3. A 25kw, 250 w dc such generator has armature and field resistance of 0.06 Ω and 100 Ω respectively. Determine the total armature power developed when working (1) as generator delivering 25 kw output and (2) as a motor taking 25 kw input.

Voltage equation of dc motor

For a generator, generated emf has to supply armature resistance drop and remaining part is available across the load as a terminal voltage. But in case of dc motor, supply voltage V has to overcome back emf E_b which is opposing V and also various drops are armature resistance drop $I_a R_a$, brush drop etc. In fact the electrical work done in overcoming the back emf gets converted into the mechanical energy, developed in the armature.

Hence, the voltage equation of a dc motor is

$$V = E_b + I_a R_a + \text{brush drops}$$

$$\text{Or } V = E_b + I_a R_a \quad \text{neglecting brush drops}$$

The back emf is always less than supply voltage ($E_b < V$) but R_a is very small hence under normal running conditions, the difference between back emf and supply voltage is very small. The net voltage across the armature is the difference between the supply voltage and back emf which decides the armature current. Hence from the voltage equation we can write $I_a = (V - E_b) / R_a$.

1. A 220 V dc motor has an armature resistance of 0.75Ω it is drawing an armature current of 30 A, during a certain load, calculate the induced emf in the motor under this condition.

$$V = 220 \text{ V}, I_a = 30 \text{ A}, R_a = 0.75\Omega$$

$$\text{For a motor, } V = E_b + I_a R_a$$

$$E_b = 197.5 \text{ V}$$

This is the induced emf called back emf in a motor.

2. A 4-pole dc motor has lap connected armature winding. The number of armature conductors is 250. When connected to 230 V dc supply it draws an armature current of 40 A calculate the back emf and the speed with which motor is running. Assume armature resistance is 0.6Ω

$$P = 4 \quad A = P = 4 \quad \text{as lap connected}$$

$$\phi = 30 \text{ m wb} = 30 \times 10^{-3} \text{ Wb}, V = 230 \text{ V}, z = 250$$

$$I_a = 40 \text{ A}$$

$$\text{From voltage equation } V = E_b + I_a R_a$$

$$230 = E_b + 40 \times 0.6$$

$$E_b = \phi P n z / 60 A$$

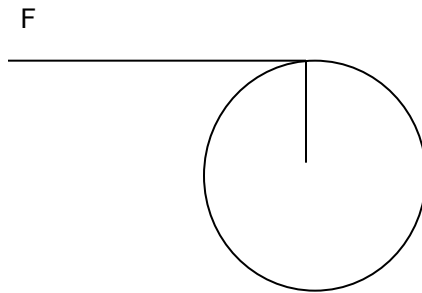
$$206 = (30 \times 10^{-3} \times 4 \times N \times 250) / (60 \times 4)$$

$$N = 1648 \text{ rpm.}$$

Torque: The turning or twisting movement of a body is called Torque.

(Or)

It is defined as the product of force and perpendicular distance $\check{T} = F \cdot R$



In case of DC motor torque is produced by the armature and shaft called as armature torque(T_a) and shaft torque(T_{sh}).

Let, N be the speed of the armature in RPM

R be the radius of the armature

Power = Work Done / Time

Work Done = Force \times Distance

The distance travelled in rotating the armature for one time = $2\pi R$

If N rotations are made in 60 sec

Then time taken for one rotation is = $60/N$

So, Power = $(F \cdot 2\pi R) / (60/N)$

$$P = \check{T} \omega$$

Here $P = E_b I_a$

But $E_b = \frac{\phi Z N P}{60 A}$

$(\frac{\phi Z N P}{60 A}) I_a = \check{T} \omega$

$$= \check{T}_a (2\pi N) / 60$$

$$\check{T}_a = 0.159 \phi Z I_a P / A$$

Similarly, Shaft torque $T_{sh} = \text{output} / \omega$

$$T_{sh} = \text{output} / ((2\pi N) / 60)$$

$$T_{sh} = 9.55 (\text{output}) / N$$

DC Motor characteristics

1. Torque and armature current (T_a/I_a characteristics)
It is also known as electrical characteristics.
2. Speed and armature current i.e., (N/T_a characteristics). It is also known as mechanical characteristics.

While discussing motor characteristics, the following two relations should always be kept in mind.

$$T_a \propto \phi I_a \text{ \& } N \propto E_b/\phi.$$

Characteristics of series motors

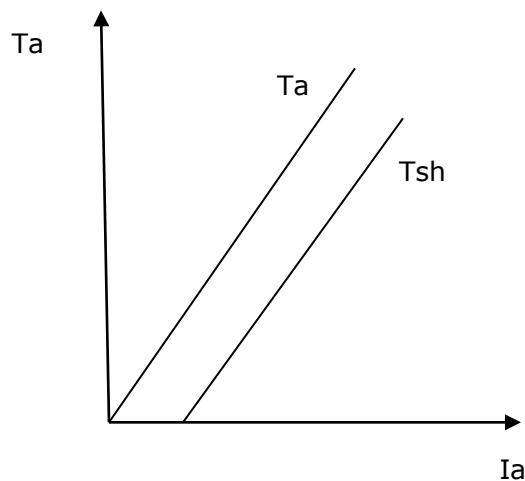
1. T_a/I_a Characteristics.

For series motor $\phi \propto I_a$

$$T_a \propto \phi I_a$$

$$\propto I_a^2$$

Thus, torque in case of series motor is proportional to the square of the armature current



As the load increases, armature current increases and torque produced increases proportional to the square of the armature current upto a certain limit.

Hence T_a/I_a curve is a parabola.

After saturation, ϕ is almost independent I_a , hence $T_a \propto I_a$ only.

So, the characteristic becomes a straight line.

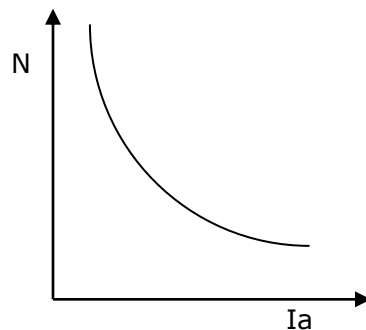
The shaft torque T_{sh} is less than the armature torque due to stray losses. It is shown dotted in the figure.

On heavy loads, a series motor events a torque proportional to the square of armature current.

Hence, where large starting torque is required for accelerating heavy masses quickly as in hasps and electric trains etc. series motors are used.

2. N/I_a characteristic $N \propto (E_b/\phi) \propto [V - I_a (R_a + R_{se})/I_a]$

The values R_a and R_{se} are so small as $\phi \propto I_a$ in case of series motor that the effect of change in I_a on speed avoid the effect of change in $V - I_a R_a - I_a R_{se}$ on speed change in E_b for various load currents is small and hence may be neglected.



With increased I_a , ϕ also increases. Hence, speed varies inversely as armature current. When load is heavy, I_a is large. Hence speed is low (this increases E_b and allows more armature current to flow). But when load current and hence I_a falls to a small value speed becomes dangerously high.

Hence, a series motor should never be started without some mechanical load on it otherwise it may develop excessive speed and get damaged due to centrifugal loss so produced. Series motor is a variable speed motor.

N/T_a characteristic:

$$T \propto I_a^2$$

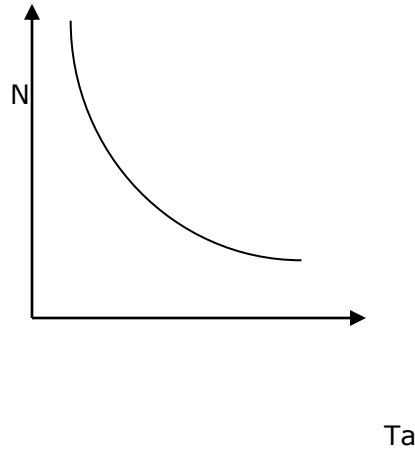
$$N \propto 1/I_a$$

$$I_a \propto \sqrt{T_a}$$

$$N \propto \frac{1}{\sqrt{T_a}}$$

$$T \propto 1/N^2$$

Thus for small T , speed is large while for large T speed is small.



As I_a increases, torque increases and speed decreases so as torque increases, speed decreases is the nature of this curve, which is similar to speed current curve.

Characteristics of shunt Motors

1. T_a/I_a characteristics:

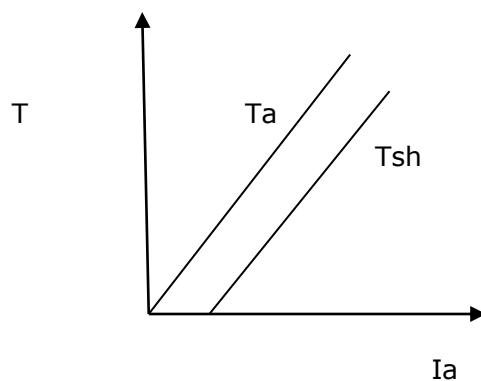
$$T \propto \phi I_a$$

For a constant value of R_{sh} and supply voltage, V , I_{sh} is also constant and hence flux is also constant.

$$T \propto I_a$$

The equation represents a straight line passing through the origin. Torque increases linearly with armature current.

It is seen earlier that armature current is decided by the load. So as load increases, armature current increases, increasing the torque developed linearly.



Now, if shaft torque is plotted against armature current, it is known that shaft torque is less than the armature torque and the difference between the two is lost torque T_L .

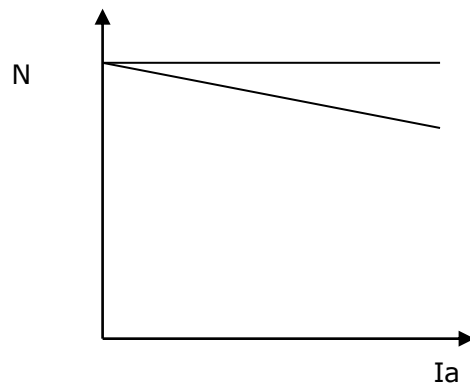
Since a heavy starting load will need a heavy starting current, shunt motor should never be started on large load.

2. **N/Ia characteristic:**

If ϕ is assumed constant, $N \propto E_b$.

As E_b is also practically constant, speed is constant

But strictly speaking, both $E_b \times \phi$ decreases with increasing load. However, E_b decreases slightly more than ϕ so that on the whole, there is some decrease in speed.



The drop varies from 5 to 15% of full-load speed, being dependent on saturation, armature reaction and brush position. Hence, the actual speed curve is slightly dropping as shown by the dotted line.

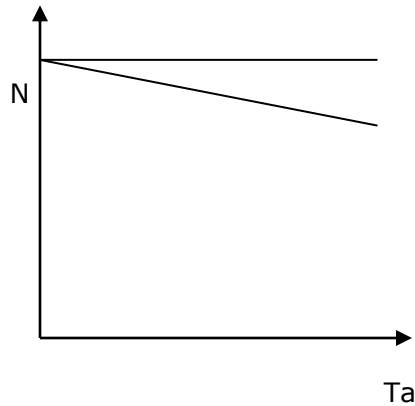
But, for all practical purposes, shunt motor is taken as a constant speed motor. Because, there is no appreciable change in the speed of a shunt motor from no-load to full-load, it may be connected to loads which are totally and student thrown off without any term of excessive speed resulting.

Since a heavy starting load will need a heavy starting current, shunt motor should never be started on heavy load.

Due to the constancy of their speed, shunt motors are suitable for driving shafting, machine tools, lathes, wood-working machines and for all other purposes where an approximately constant speed is required.

3. N/Ta characteristics

These can be deduced from the above two characteristics this graph is similar to speed-armature current characteristics as torque is proportional to the armature current. This curve shows that the speed almost remains constant through torque changes from no load to full load conditions.



Compound Motors :

Compound motor characteristics basically depends on the fact whether the motor is correlatively compound or differential compound. All the characteristics of the compound motor are the combination of the shunt and series characteristics.

Cummulative – compound motors

These are capable of developing large amount of torque at low speeds just like a series motor. However it is not having a disadvantage of series motor even at light or no load.

Due to shunt windings, speed will not become excessively high but due to series windings, it will be able to take heavy loads.

Differential compound: Here the two fluxes oppose each other, the resultant flux decreases as load increases, thus the machine runs at a higher speed with increase in the load. This property is dangerous as

The following results were obtained from a static torque tests on a series motor:

Current (A) :	20	30	40	50
	128.8	230.5	349.8	469.2

Deduce the speed/torque curve for the machine when supplied at a constant voltage of 460 V.

Resistance of armature and field winding is 0.5 Ω . Ignore iron and friction losses.

When input current is 20A, we have

Motor input = $460 \times 20 = 9,200 \text{ W}$

Field and armature cu loss = $202 \times 0.5 = 200 \text{ w}$

Ignoring iron and friction losses, output = $9,200 - 200$

= 9000 w

$T_{sh} \times 2\pi N = \text{output in watts}$

$128.8 \times 2\pi \times N = 9,000$

$N = 9000 / (2\pi \times 128.8) = 11.12 \text{ rps} = 6.667 \text{ rpm}$

Current (A)	20	30	40	50
Input (W)	9200	13800	18400	23000
$I^2 R$ loss(w)	200	450	800	1250
Output (w)	9000	13350	17600	21850
Speed (rpm)	667	551	480	445
Torque (N-m)	128.8	230.5	349.8	469.2

On full load, the motor may try to run with dangerously high speed. So, differential compound motor is generally not used in practice.

The exact shape of these characteristics depend on the relative contribution of series and shunt field windings. If the shunt field winding is more dominant, then the characteristics take the shape of the shunt motor characteristics.

While, if the series field winding is more dominant then the characteristics take the shape of series characteristic.

Type of Motor	Characteristics	Applications
Shunt	Approximately constant speed. Adjustable speed medium starting torque (up to 1.5 F. L. torque)	For driving constant speed some centrifugal pumps machine tools blows and fans reciprocating pumps
Series	Variable speed adjustable varying speed high starting torque	For traction work i.e. electric locomotives repaid transit system trolley cars etc. Cranes and hoists conveyors.
Cumulative compound	Variable speed adjustable varying speed high starting torque	For intermittent high torque load for shears and punches elevators conveyers clearly planer

2. A 500 v dc shunt motor takes a current of 5 A on no-load the resistance of the armature and field circuits are 0.22Ω and 250Ω respectively. Find (a) the efficiency when loaded and taking a current of 100A.

b) The percentage change of speed state precisely the assumptions made.

No-load condition

$$I_{sh} = 500/250 = 2A$$

$$I_{ao} = 5 - 2 = 3A$$

$$E_{bo} = 500 - (3 \times 0.22) = 4.99.34 \text{ v}$$

$$\text{Ar.cu.loss} = 3^2 \times 0.22 = 2110 \text{ W}$$

$$\text{Total losses} = 2110 + 2498 = 4608$$

$$\text{Motor input} = 500 \times 100 = 50000 \text{ W}$$

$$\text{Motor output} = 50000 - 4608$$

$$= 45392 \text{ W}$$

$$\eta = 45392/50000 = 0.908 \text{ } 90.8\%$$

$$N/N_o = E_b/E_{bo} = 478.44/499.34 \text{ or}$$

$$(N-N_o)/N_o = -20.9/499.34 = -0.0418 \text{ or } -4.18\%$$

4. The armature winding of a 4-pole, 250-v, dc shunt motor is lap connected there are 120 slots, each slot containing 8 conductors. The flux per pole is 20 m wb and the current taken by the motor is 25 A. The resistance of armature and field circuit are 0.1Ω and 125Ω respectively.

5. If the rotational loose amount to 810w, find the developed torque and useful torque of the machine.

$$I_a = 25 - 2 = 23 \text{ A}$$

$$E_b = 250 - (23 \times 0.1) = 247.7 \text{ v}$$

$$E_b = f \phi z n / 60 \text{ (p/A)} \quad 247.7 = (2 \times 10^{-3} \times 960 \times N) / 60 = (4/4)$$

$$\begin{aligned} \text{A} \quad a &= 9.55 (E_b I_a) / N \quad N = 773 \text{ rpm} \\ &= 9.55 \times (247.7 \times 23) / 773 = 70.4 \text{ N-m} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad \text{Ar cu loss} &= 232 \times 0.1 = 53 \text{ w} \\ \text{shunt cu loss} &= 250 \times 2 = 500 \text{ w} \\ \text{Rotational loses} &= 810 \text{ w} \\ \text{Total motor losses} &= 1363 \text{ w} \\ \text{Motor input} &= 250 \times 25 = 6250 \text{ w} \\ \text{Motor output} &= 6.250 - 1363 = 4.887 \text{ w} \\ T_{sh} &= 9.55 \times 4.887 / 773 = 60.4 \text{ N-m} \end{aligned}$$

Speed control of DC motors:

The speed can be controlled by varying

1. flux/pole, ϕ (flux control)
2. resistance R_a of armature circuit (Rheistac control)

Speed control of shunt motors

1. Variation of flux or flux control method

$$N \propto 1/\phi.$$

By decreasing the flux, the speed can be increased and vice versa. Hence the name flux or field control method. The flux of a dc motor can be changed by changing I_{sh} with the help of a shunt field rheostat.

1. A 250-v dc shunt motor has armature resistance of 0.25Ω on load it takes an armature current of 50A and runs at 750 rpm. If the flux of motor is reduced by 10% without changing the load torque, find the new speed of the motor.

$$\begin{aligned} N_2/N_1 &= E_{b2}/E_{b1} \times \phi_1/\phi_2 & T_a &\propto \phi I_a \\ \text{Hence } T_{a1} &\propto \phi_1 I_{a1} \\ T_{a2} &\propto \phi_2 I_{a2} \end{aligned}$$

$$\text{Since } T_{a1} = T_{a2} \quad \phi T_{a1} = \phi_2 I_{a2}.$$

$$\begin{aligned} \phi_2 &= 0.9\phi_1 \\ 50 \phi_1 &= 0.9\phi_1 I_{a2} \\ I_{a2} &= 55.6 \text{ A} \end{aligned}$$

$$\begin{aligned} E_{b1} &= 250 - (50 \times 0.25) \\ &= 231.1 \text{ v} \\ N_2/750 &= 231.1/237.5 \times \phi_1/0.9\phi_1 \\ N_2 &= 81 \text{ rpm} \end{aligned}$$

2. A 220 v shunt motor has an armature resistance of By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant.

$$\begin{aligned} N_2/N_1 &= E_{b2}/E_{b1} \times \phi_1/\phi_2 \\ \text{Since torque remains constant, hence } \phi_1 I_{a1} &= \phi_2 I_{a2} \\ I_{a2} &= I_{a1} \quad \phi_1/\phi_2 = 40x \\ \text{Where } x &= \phi_1/\phi_2 \\ E_{b1} &= 220 - (40 \times 0.5) \\ &= 200 \text{ v} \\ E_{b2} &= 220 - (40x \times 0.5) \end{aligned}$$

$$\begin{aligned}
 &= 220 - 20x \\
 N_2/N_1 &= 3/2 \text{ (given)} \\
 3/2 &= [(220 - 20x)/200] \times X \\
 x^2 - 11x + 15 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \phi_1/\phi_2 &= 1.6 & \phi_2/\phi_1 &= 1/1.6 \\
 (\phi_1 - \phi_2)/\phi_1 &= 1.6 - 1/1.6 = 3/8 \\
 \text{percentage change in flux} &= 3/8 \times 10 = 37.5\%
 \end{aligned}$$

2) Armature or rheostatic control method :

This method is used when speeds below the no-load speed are required. As the supply voltage is normally constant, the voltage across the armature is varied by inserting a variable rheostat or resistance in series with the armature circuit as shown. As controller resistance is increased, p.d. across the armature is decreased, thereby decreasing the armature speed for a load of constant torque, speed is approximately proportional to the p.d. across the armature.

From the speed/armature current characteristic it is such that greater the resistance in the armature circuit, greater is the fall in speed.

Let I_{a1} = armature current in the first case

I_{a2} = armature current in the second case

N_1, N_2 = corresponding speeds; V = supply voltage

Then $N_1 \propto V - I_{a1} R_a$

$V \propto E_{b1}$

Let some controller resistance of value R be added to the armature circuit resistance so that its value becomes $(R + R_a) = R_t$

Then $N_2 \propto V - I_{a2} R_t$

$$N_2/N_1 = E_{b2}/E_{b1}$$

$$N/N_o = (V - I_a R_t) / (V - I_{a_o} R_a)$$

Neglecting $I_{a_o} R_a$ w.r.t. V , we get

$$N = N_o (1 - I_a R_t/V)$$

For a given resistance R_t the speed is a linear function of armature current I_a .

The load current for which the speed would be zero is found by putting $N = 0$ in the above relation

$$0 = N_o (1 - i_a R_t/V)$$

$$I_a = V/R_t$$

This is the maximum current known as stalling current.

Losses in a DC motor: Losses in a DC Motor are similar to that of a DC Generator